Homework 2
CS 3385

1. Consider the function $f(x) = x^3$.
   (a) (4 points) In what range is $f(x)$ monotonically increasing?
   (b) (4 points) In what range is $f(x)$ monotonically decreasing?

2. Let $f(n)$ and $g(n)$ be monotonically increasing functions. Show that each of the following functions are also monotonically increasing. You'll do this using the definition of monotonicity.
   (a) (4 points) $f(n) + g(n)$
   (b) (4 points) $f(g(n))$
   (c) (4 points) $f(n) \cdot g(n)$ (Assume that $f(n)$ and $g(n)$ are both nonnegative.)

3. (10 points) Prove equation 3.19 from the book. Stirling's approximation and the various properties of logarithms will be helpful.

4. (8 points) Let

   $$p(n) = \Sigma_{i=0}^{d} a_i n^i$$

   where $a_d > 0$, be a degree-d polynomial in n, and let k be a constant such that $k \geq d$. Show that $p(n) = O(n^k)$. You will use the definition of $O(g(n))$.

5. Consider $\left(\frac{3}{2}\right)^n = \Omega\left(2^{\lg n}\right)$.
   (a) (5 points) Show that the equality is true.
   (b) (5 points) What are all valid values of c when $n_0 = 1$?

6. Consider $n! = O((n + 1)!)$.
   (a) (5 points) Show that the equality is true.
   (b) (5 points) What are all valid values of n_0 when $c = 1$?

7. Let $f(n)$ and $g(n)$ be asymptotically positive functions. Prove or disprove each of the following conjectures.
   (a) (5 points) $f(n) = O(g(n))$ implies $g(n) = O(f(n))$.
   (b) (5 points) $f(n) = O((f(n))^2)$.
   (c) (5 points) $f(n) = O(g(n))$ implies $g(n) = \Omega(f(n))$.
   (d) (5 points) $f(n) = \Theta(f(n/2))$.

8. Let $f(x) = \frac{x^2}{2}$. Give the following values. Show your work.
   (a) (3 points) $f^3(1)$
   (b) (3 points) $f^3(2)$
   (c) (3 points) $f^3(3)$

9. (13 points) Rank the following functions by order of growth, slowest to fastest; that is, find an arrangement $g_1, g_2, \ldots$ of the functions satisfying $g_1 = O(g_2), g_2 = O(g_3), \ldots$. In your ordering, circle any functions that have the same asymptotic rate of growth.

$n2^n$	n	$\lg \lg n$	n^3	$4^{\lg n}$
$n \lg n$	$\sqrt{n}$	$(3/2)^n$	$2^{\lg n}$	
2^n	n^2	$\lg n$	$n!$	