# CS 4499/5599 HW 6

## 1 Probability

Below is a table listing the probabilities of three binary random variables. In the empty table cells, fill in the correct values for each marginal or conditional probability. Your answers should to 4 decimal places.

| $X_0$ | $X_1$ | $X_2$ | $P(◀,▶)$ |
|---|---|---|---|
| 0 | 0 | 0 | 0.100 |
| 1 | 0 | 0 | 0.140 |
| 0 | 1 | 0 | 0.140 |
| 1 | 1 | 0 | 0.140 |
| 0 | 0 | 1 | 0.140 |
| 1 | 0 | 1 | 0.100 |
| 0 | 1 | 1 | 0.140 |
| 1 | 1 | 1 | 0.100 |

| Expression | Value |
|---|---|
| $P(X_0 = 1, X_1 = 0, X_2 = 1)$ | |
| $P(X_0 = 0, X_1 = 1)$ | |
| $P(X_2 = 0)$ | |
| $P(X_1 = 0 \mid X_0 = 1)$ | |
| $P(X_0 = 1, X_1 = 0 \mid X_2 = 1)$ | |
| $P(X_0 = 1 \mid X_1 = 0, X_2 = 1)$ | |

# 2 Joint Distributions

You are given the prior distribution P(X), and two conditional distributions P(Y|X) and P(Z|Y) as below (you are also given the fact that Z is independent from X given Y). All variables are binary variables. Compute the following joint distributions based on the chain rule. Your answers should be to 4 decimal places.

| $X$ | $P(X)$ |
|---|---|
| 0 | 0.900 |
| 1 | 0.100 |

| $Y$ | $X$ | $P(Y\|X)$ |
|---|---|---|
| 0 | 0 | 0.600 |
| 1 | 0 | 0.400 |
| 0 | 1 | 0.300 |
| 1 | 1 | 0.700 |

| $Z$ | $Y$ | $P(Z\|Y)$ |
|---|---|---|
| 0 | 0 | 0.400 |
| 1 | 0 | 0.600 |
| 0 | 1 | 0.300 |
| 1 | 1 | 0.700 |

| $X$ | $Y$ | $P(X,Y)$ |
|---|---|---|
| 0 | 0 | |
| 1 | 0 | |
| 0 | 1 | |
| 1 | 1 | |

| $X$ | $Y$ | $Z$ | $P(X,Y,Z)$ |
|---|---|---|---|
| 0 | 0 | 0 | |
| 1 | 0 | 0 | 0.012 |
| 0 | 1 | 0 | 0.108 |
| 1 | 1 | 0 | |
| 0 | 0 | 1 | 0.324 |
| 1 | 0 | 1 | |
| 0 | 1 | 1 | 0.252 |
| 1 | 1 | 1 | |

# 3 Independence Assumptions

For each of the following four subparts, you are given three joint probability distribution tables. For each distribution, please identify if the given independence / conditional independence assumption is true or false. For your convenience, we have also provided some marginal and conditional probability distribution tables that could assist you in solving this problem.

a) X is independent from Y

| $X$ | $Y$ | $P(X,Y)$ |
|---|---|---|
| 0 | 0 | 0.040 |
| 1 | 0 | 0.060 |
| 0 | 1 | 0.360 |
| 1 | 1 | 0.540 |

| $X$ | $P(X)$ |
|---|---|
| 0 | 0.400 |
| 1 | 0.600 |

| $X$ | $Y$ | $P(X\|Y)$ |
|---|---|---|
| 0 | 0 | 0.400 |
| 1 | 0 | 0.600 |
| 0 | 1 | 0.400 |
| 1 | 1 | 0.600 |

b) X is independent from Y

| $X$ | $Y$ | $P(X,Y)$ |
|---|---|---|
| 0 | 0 | 0.220 |
| 1 | 0 | 0.180 |
| 0 | 1 | 0.300 |
| 1 | 1 | 0.300 |

| $Y$ | $P(Y)$ |
|---|---|
| 0 | 0.400 |
| 1 | 0.600 |

| $X$ | $P(X)$ |
|---|---|
| 0 | 0.520 |
| 1 | 0.480 |

c) X is independent from Y given Z

| $X$ | $Y$ | $Z$ | $P(X,Y,Z)$ |
|---|---|---|---|
| 0 | 0 | 0 | 0.128 |
| 1 | 0 | 0 | 0.032 |
| 0 | 1 | 0 | 0.032 |
| 1 | 1 | 0 | 0.008 |
| 0 | 0 | 1 | 0.120 |
| 1 | 0 | 1 | 0.280 |
| 0 | 1 | 1 | 0.120 |
| 1 | 1 | 1 | 0.280 |

| $X$ | $Z$ | $P(X\|Z)$ |
|---|---|---|
| 0 | 0 | 0.800 |
| 1 | 0 | 0.200 |
| 0 | 1 | 0.300 |
| 1 | 1 | 0.700 |

| $X$ | $Y$ | $Z$ | $P(X\|Y,Z)$ |
|---|---|---|---|
| 0 | 0 | 0 | 0.800 |
| 1 | 0 | 0 | 0.200 |
| 0 | 1 | 0 | 0.800 |
| 1 | 1 | 0 | 0.200 |
| 0 | 0 | 1 | 0.300 |
| 1 | 0 | 1 | 0.700 |
| 0 | 1 | 1 | 0.300 |
| 1 | 1 | 1 | 0.700 |

d) X is independent from Y given Z

| $X$ | $Y$ | $Z$ | $P(X,Y,Z)$ |
|---|---|---|---|
| 0 | 0 | 0 | 0.070 |
| 1 | 0 | 0 | 0.070 |
| 0 | 1 | 0 | 0.420 |
| 1 | 1 | 0 | 0.140 |
| 0 | 0 | 1 | 0.030 |
| 1 | 0 | 1 | 0.150 |
| 0 | 1 | 1 | 0.030 |
| 1 | 1 | 1 | 0.090 |

| $X$ | $Z$ | $P(X\|Z)$ |
|---|---|---|
| 0 | 0 | 0.700 |
| 1 | 0 | 0.300 |
| 0 | 1 | 0.200 |
| 1 | 1 | 0.800 |

| $X$ | $Y$ | $Z$ | $P(X\|Y,Z)$ |
|---|---|---|---|
| 0 | 0 | 0 | 0.500 |
| 1 | 0 | 0 | 0.500 |
| 0 | 1 | 0 | 0.750 |
| 1 | 1 | 0 | 0.250 |
| 0 | 0 | 1 | 0.167 |
| 1 | 0 | 1 | 0.833 |
| 0 | 1 | 1 | 0.250 |
| 1 | 1 | 1 | 0.750 |

# 4 Chain Rule

a) Given no independence assumptions, which of the following are equivalent to *P(A,B|C)*? Indicate all that apply.

- ☐ $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- ☐ $\frac{P(B,C|A)P(A)}{P(B,C)}$
- ☐ $P(A|B,C)\,P(B|C)$
- ☐ $\frac{P(A|C)P(B,C)}{P(C)}$

b) Given that A is independent of B given C, which of the following are equivalent to *P(A,B|C)*? Indicate all that apply.

- ☐ $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- ☐ $\frac{P(B,C|A)P(A)}{P(B,C)}$
- ☐ $P(A|B,C)\,P(B|C)$
- ☐ $\frac{P(A|C)P(B,C)}{P(C)}$

c) Given no independence assumptions, which of the following are equivalent to *P(A|B, C)*? Indicate all that apply.

- [ ] $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [ ] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [ ] $\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$
- [ ] $\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$

d) Given that A is independent of B given C, which of the following are equivalent to P(A|B, C)? Indicate all that apply.

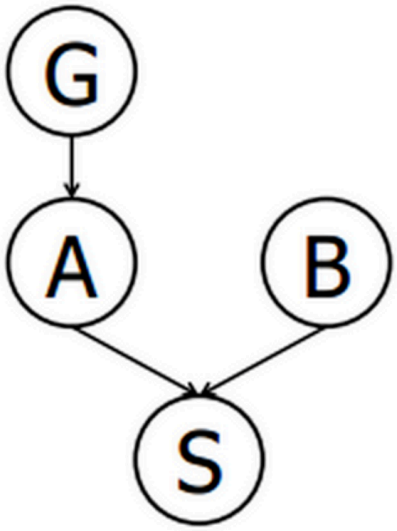
- [ ] $\frac{P(C|A)P(A|B)P(B)}{P(C)}$
- [ ] $\frac{P(B,C|A)P(A)}{P(B,C)}$
- [ ] $\frac{P(A|C)P(C|B)P(B)}{P(B,C)}$
- [ ] $\frac{P(C|A,B)P(B|A)P(A)}{P(B|C)P(C)}$

# 5 Bayes' Nets

Suppose that a patient can have a symptom (S) that can be caused by two different diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A. The Bayes' Net and corresponding probability tables for this situation are shown below.

| $G$ | $P(G)$ |
|---|---|
| $g$ | 0.10 |
| $\neg g$ | 0.90 |

| $B$ | $P(B)$ |
|---|---|
| $b$ | 0.40 |
| $\neg b$ | 0.60 |




| $G$ | $A$ | $P(A \mid G)$ |
|---|---|---|
| $g$ | $a$ | 1.00 |
| $g$ | $\neg a$ | 0.00 |
| $\neg g$ | $a$ | 0.10 |
| $\neg g$ | $\neg a$ | 0.90 |

| $A$ | $B$ | $S$ | $P(S \mid A, B)$ |
|---|---|---|---|
| $a$ | $b$ | $s$ | 1.00 |
| $a$ | $b$ | $\neg s$ | 0.00 |
| $a$ | $\neg b$ | $s$ | 0.90 |
| $a$ | $\neg b$ | $\neg s$ | 0.10 |
| $\neg a$ | $b$ | $s$ | 0.80 |
| $\neg a$ | $b$ | $\neg s$ | 0.20 |
| $\neg a$ | $\neg b$ | $s$ | 0.10 |
| $\neg a$ | $\neg b$ | $\neg s$ | 0.90 |

a) What is P(g, a, b, s)?

b) What is the probability that a patient has disease A?

c) What is the probability that a patient has disease A given that they have disease B?

d) What is the probability that a patient has disease A given that they have symptom S and disease B?

e) What is the probability that a patient has the disease carrying gene variation G given that they have disease A?

f) What is the probability that a patient has the disease carrying gene variation G given that they have disease B?